NAME ______________________ DATE __________ PERIOD ____

10-6 Study Guide and Intervention

Exponential Growth and Decay

Exponential Decay Depreciation of value and radioactive decay are examples of **exponential decay**. When a quantity decreases by a fixed percent each time period, the amount of the quantity after t time periods is given by $y = a(1 - r)^t$, where a is the initial amount and r is the percent decrease expressed as a decimal.

Another exponential decay model often used by scientists is $y = ae^{-kt}$, where k is a constant.

Example **CONSUMER PRICES As technology advances, the price of many technological devices such as scientific calculators and camcorders goes down. One brand of hand-held organizer sells for \$89.**

a. If its price decreases by 6% per year, how much will it cost after 5 years?

Use the exponential decay model with initial amount \$89, percent decrease 0.06, and time 5 years.

$y = a(1 - r)^t$	Exponential decay formula
$y = 89(1 - 0.06)^5$	$a = 89$, $r = 0.06$, $t = 5$
$y = \$65.32$	

After 5 years the price will be \$65.32.

b. After how many years will its price be \$50?

To find when the price will be \$50, again use the exponential decay formula and solve for t.

$y = a(1 - r)^t$	Exponential decay formula
$50 = 89(1 - 0.06)^t$	$y = 50$, $a = 89$, $r = 0.06$
$\frac{50}{89} = (0.94)^t$	Divide each side by 89.
$\log\left(\frac{50}{89}\right) = \log\,(0.94)^t$	Property of Equality for Logarithms
$\log\left(\frac{50}{89}\right) = t \log 0.94$	Power Property
$t = \dfrac{\log\left(\frac{50}{89}\right)}{\log 0.94}$	Divide each side by log 0.94.
$t \approx 9.3$	

The price will be \$50 after about 9.3 years.

Exercises

1. **BUSINESS** A furniture store is closing out its business. Each week the owner lowers prices by 25%. After how many weeks will the sale price of a \$500 item drop below \$100?

$100 = 500(1-.25)^t$ $\quad \frac{\log .2}{\log .75} = t \quad t = 5.6 \quad$ 6wks

$.2 = .75^t$

CARBON DATING Use the formula $y = ae^{-0.00012t}$, where a is the initial amount of Carbon-14, t is the number of years ago the animal lived, and y is the remaining amount after t years.

2. How old is a fossil remain that has lost 95% of its Carbon-14? $.05 = e^{-.00012t}$ 24,964 yrs old

3. How old is a skeleton that has 95% of its Carbon-14 remaining?

$.95 = e^{-.00012t}$

427.4 yrs

10-6 Study Guide and Intervention *(continued)*

Exponential Growth and Decay

Exponential Growth Population increase and growth of bacteria colonies are examples of **exponential growth.** When a quantity increases by a fixed percent each time period, the amount of that quantity after t time periods is given by $y = a(1 + r)^t$, where a is the initial amount and r is the percent increase (or rate of growth) expressed as a decimal.

Another exponential growth model often used by scientists is $y = ae^{kt}$, where k is a constant.

Example **A computer engineer is hired for a salary of \$28,000. If she gets a 5% raise each year, after how many years will she be making \$50,000 or more?**

Use the exponential growth model with $a = 28{,}000$, $y = 50{,}000$, and $r = 0.05$ and solve for t.

$y = a(1 + r)^t$	Exponential growth formula
$50{,}000 = 28{,}000(1 + 0.05)^t$	$y = 50{,}000$, $a = 28{,}000$, $r = 0.05$
$\frac{50}{28} = (1.05)^t$	Divide each side by 28,000.
$\log\left(\frac{50}{28}\right) = \log (1.05)^t$	Property of Equality of Logarithms
$\log\left(\frac{50}{28}\right) = t \log 1.05$	Power Property
$t = \dfrac{\log\left(\frac{50}{28}\right)}{\log 1.05}$	Divide each side by log 1.05.
$t \approx 11.9$ years	Use a calculator.

If raises are given annually, she will be making over \$50,000 in 12 years.

Exercises

A-Perk

1. **BACTERIA GROWTH** A certain strain of bacteria grows from 40 to 326 in 120 minutes. Find k for the growth formula $y = ae^{kt}$, where t is in minutes. $326 = 40e^{120k}$ $8.15 = e^{120k}$ $\ln 8.15 = 120k$ $k = .01748$

2. **INVESTMENT** Carl plans to invest \$500 at 8.25% interest, compounded continuously. How long will it take for his money to triple? $3 = e^{.0825t}$ $\ln 3 = .0825t$ $13.3 = t$

3. **SCHOOL POPULATION** There are currently 850 students at the high school, which represents full capacity. The town plans an addition to house 400 more students. If the school population grows at 7.8% per year, in how many years will the new addition be full? $1250 = 850(1.078)^t$ $1.4706 = 1.078^t$ $t = 5.13$ yrs

4. **EXERCISE** Hugo begins a walking program by walking $\frac{1}{2}$ mile per day for one week. Each week thereafter he increases his mileage by 10%. After how many weeks is he walking more than 5 miles per day? $5 = \frac{1}{2}(1.10)^t$ $10 = 1.10^t$ $t = 24.2$ weeks

5. **VOCABULARY GROWTH** When Emily was 18 months old, she had a 10-word vocabulary. By the time she was 5 years old (60 months), her vocabulary was 2500 words. If her vocabulary increased at a constant percent per month, what was that increase?

$2500 = 10(1+r)^{42}$

$.14 = r$

14%

NAME Key DATE ________ PERIOD ____

10-6 Skills Practice

Exponential Growth and Decay

$y = a(1+r)^t$

$y = ae^{-kt}$

Solve each problem.

1. **FISHING** In an over-fished area, the catch of a certain fish is decreasing at an average rate of 8% per year. If this decline persists, how long will it take for the catch to reach half of the amount before the decline?

   $\frac{1}{2} = (\cancel{1.08}\ .92)^t$ $\log .5 = t(\log \cancel{1.08}\ .92)$

   8.31 yrs ≈ t

2. **INVESTING** Alex invests \$2000 in an account that has a 6% annual rate of growth. To the nearest year, when will the investment be worth \$3600?

   $3600 = 2000(1.06)^t$

   $1.8 = 1.06^t$ $t = 10.08$ 10 yrs

3. **POPULATION** A current census shows that the population of a city is 3.5 million. Using the formula $P = ae^{rt}$, find the expected population of the city in 30 years if the growth rate r of the population is 1.5% per year, a represents the current population in millions, and t represents the time in years.

   $P = 3.5e^{(.015)30}$

   P = 5.49 mill

4. **POPULATION** The population P in thousands of a city can be modeled by the equation $P = 80e^{0.015t}$, where t is the time in years. In how many years will the population of the city be 120,000?

   $120 = 80e^{.015t}$ $\ln 1.5 = .015t$

   $1.5 = e^{.015t}$ 27.0 ≈ t yrs

5. **BACTERIA** How many days will it take a culture of bacteria to increase from 2000 to 50,000 if the growth rate per day is 93.2%?

   $50,000 = 2,000(1.932)^t$

   $25 = 1.932^t$ t = 4.89 days

6. **NUCLEAR POWER** The element plutonium-239 is highly radioactive. Nuclear reactors can produce and also use this element. The heat that plutonium-239 emits has helped to power equipment on the moon. If the half-life of plutonium-239 is 24,360 years, what is the value of k for this element?

   $\frac{1}{2} = e^{-k(24,360)}$ $.00002845 \times 10^{-5}$

   k = .000028

   $\ln \frac{1}{2} = -24,360k$

7. **DEPRECIATION** A Global Positioning Satellite (GPS) system uses satellite information to locate ground position. Abu's surveying firm bought a GPS system for \$12,500. The GPS depreciated by a fixed rate of 6% and is now worth \$8600. How long ago did Abu buy the GPS system?

   $8600 = 12,500(1-.06)^t$ [redacted]

   $.688 = .94^t$ .688

   6.04 ≈ t 6 yrs ago

8. **BIOLOGY** In a laboratory, an organism grows from 100 to 250 in 8 hours. What is the hourly growth rate in the growth formula $y = a(1+r)^t$?

   $250 = 100(1+r)^8$

   $(2.5)^{1/8} = ((1+r)^8)^{1/8}$

   $1.1214 = 1+r$

   $.1214 = r$

   12.1%

Lesson 10-6

Chapter 10

NAME ______________________ DATE ____________ PERIOD ______

10-6 Practice

Exponential Growth and Decay

Solve each problem.

1. **INVESTING** The formula $A = P\left(1 + \frac{r}{2}\right)^{2t}$ gives the value of an investment after t years in an account that earns an annual interest rate r compounded twice a year. Suppose \$500 is invested at 6% annual interest compounded twice a year. In how many years will the investment be worth \$1000?

$1000 = 500\left(1+\frac{.06}{2}\right)^{2t}$ $\quad \log 2 = 2t \log 1.03$

$2 = 1.03^{2t}$ $\quad 11.7 = t$ yrs

2. **BACTERIA** How many hours will it take a culture of bacteria to increase from 20 to 2000 if the growth rate per hour is 85%?

$2000 = 20(1.85)^t$

$100 = 1.85^t$ $\quad t = 7.5$ hrs

3. **RADIOACTIVE DECAY** A radioactive substance has a half-life of 32 years. Find the constant k in the decay formula for the substance.

$\frac{1}{2} = e^{-k32}$

$k = .02166$

4. **DEPRECIATION** A piece of machinery valued at \$250,000 depreciates at a fixed rate of 12% per year. After how many years will the value have depreciated to \$100,000?

$100{,}000 = 250{,}000(1-.12)^t$

$\log .4 = t \log .88$ $\quad t = 7.16$ yrs

Oops 5. **INFLATION** For Dave to buy a new car comparably equipped to the one he bought 8 years ago would cost \$12,500. Since Dave bought the car, the inflation rate for cars like his has been at an average annual rate of 5.1%. If Dave originally paid \$8400 for the car, how long ago did he buy it?

$12{,}500 = 8400(1.051)^t$

$1.488 = 1.051^t$ $\quad$ 8yrs

$8 = t$

6. **RADIOACTIVE DECAY** Cobalt, an element used to make alloys, has several isotopes. One of these, cobalt-60, is radioactive and has a half-life of 5.7 years. Cobalt-60 is used to trace the path of nonradioactive substances in a system. What is the value of k for Cobalt-60?

$\frac{1}{2} = e^{-k(5.7)}$ $\quad k = .1216$

7. **WHALES** Modern whales appeared 5–10 million years ago. The vertebrae of a whale discovered by paleontologists contain roughly 0.25% as much carbon-14 as they would have contained when the whale was alive. How long ago did the whale die? Use $k = 0.00012$.

$.0025 = e^{-.00012t}$

$t = 49{,}928$ yrs $\quad \approx 50{,}000$

8. **POPULATION** The population of rabbits in an area is modeled by the growth equation $P(t) = 8e^{0.26t}$, where P is in thousands and t is in years. How long will it take for the population to reach 25,000?

$25 = 8e^{.26t}$

$3.125 = e^{.26t}$ $\quad t = 4.38$ yrs

9. **DEPRECIATION** A computer system depreciates at an average rate of 4% per month. If the value of the computer system was originally \$12,000, in how many months is it worth \$7350?

$7350 = 12000(1-.04)^t$

$.6125 = .96^t$ $\quad$ 12months

10. **BIOLOGY** In a laboratory, a culture increases from 30 to 195 organisms in 5 hours. What is the hourly growth rate in the growth formula $y = a(1 + r)^t$?

$195 = 30(1+r)^5$

45.4%

$(6.5)^{\frac{1}{5}} = (1+r)^{5 \cdot \frac{1}{5}}$

$.454 = r$